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# BOOLEAN MODEL OF HSUH<sup>1</sup> LEAKAGE TESTING FOR THE PURPOSE OF DESIGNING A DIAGNOSING UNIT OF SELECTED DAMAGES OF VEHICLE BRAKING MECHANISMS

# MODEL BOOLE'OWSKI BADANIA SZCZELNOŚCI HSUH<sup>2</sup> NA POTRZEBY ZAPROJEKTOWANIA DIAGNOZERA WYBRANYCH USZKODZEŃ MECHANIZMÓW HAMULCOWYCH POJAZDÓW

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## Summary

The technical condition of the braking system of a vehicle admitted to traffic on public roads cannot raise any objections. Regardless of the intended use and structural solution, the braking systems can be divided into braking mechanisms and mechanisms and systems activating braking. The technical diagnostics include the assessment of technical suitability of the mechanisms activating braking and the determination of the braking system's performance at the test and measurement

<sup>&</sup>lt;sup>1</sup> HSUH – Hydraulic Vehicle Braking System

<sup>&</sup>lt;sup>2</sup> HSUH – Hydrauliczny Samochodowy Układ Hamulcowy

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stand. For a diagnostician who checks the technical condition of a vehicle, this is a basic check in the field of road use safety. One of the symptoms of the technical failure of the vehicle braking system is the lack of tightness of the hydraulic circuit. In the process of exploitation, taking into account the safety of technical facilities, the factor that influences the development of diagnostics involves primarily the minimization of threats to human health and life, threats to the biological and technical environment. Guided by these assumptions, the purpose of the conducted research was to develop a method for diagnosing leakage of the hydraulic circuit for the needs of designing a braking system diagnosing unit. The publication presents the original mathematical model based on Boolean functions, which is a part of the mathematical analyses.

**Keywords:** braking system, hydraulic circuit, operation, leakage, mathematical modelling, Boolean functions

## Streszczenie

Stan techniczny układu hamulcowego pojazdu dopuszczonego do ruchu na drogach publicznych nie może budzić zastrzeżeń. Niezależnie od przeznaczenia i rozwiązania konstrukcyjnego układy hamulcowe można podzielić na mechanizmy hamulcowe

i mechanizmy oraz systemy uruchamiające hamulce. Diagnostyka techniczna obejmuje ocenę zdatności technicznej mechanizmów uruchamiających hamulce oraz określanie skuteczności działania układu hamulcowego na stanowisku badawczo-pomiarowym. Dla diagnosty, dokonującego sprawdzenia stanu technicznego pojazdu, jest to zasadnicze badanie kontrolne

w zakresie bezpieczeństwa użytkowania pojazdu w ruchu drogowym. Jednym z symptomów niezdatności technicznej samochodowego układu hamulcowego jest brak szczelności obwodu hydraulicznego. W procesie eksploatacji mając na uwadze bezpieczeństwo obiektów technicznych czynnikiem wpływającym na rozwój diagnostyki jest przede wszystkim minimalizacja zagrożeń zdrowia i życia ludzkiego, zagrożeń środowiska biologicznego i technicznego. Kierując się tymi założeniami celem przeprowadzonych badań było opracowanie metody procesu diagnozowania nieszczelności obiegu hydraulicznego na potrzeby zaprojektowania diagnozera mechanizmów hamulcowych. W publikacji przedstawiono Autorski model matematyczny oparty na funkcjach boole'owskich, będący częścią prowadzonych analiz matematycznych.

**Słowa kluczowe:** układ hamulcowy, obieg hydrauliczny, eksploatacja, nieszczelność, modelowanie matematyczne, funkcje boole'owskie

## **1. Introduction**

The development and design changes of motor vehicles cause a lot of progressin the field of diagnosis. The research methods have changed, modern stations equipped with the latest generation of control and measurement devices have been created, thanks to which we can reliably test technical facilities, including braking systems of various types. Correct and safe braking is the sum of a few basic elements [22]:

- 1) properly designed braking system construction for a given type of vehicle,
- 2) correct selection of friction materials for a given type of braking system and vehicle,
- 3) adequate quality and execution of individual components of the braking mechanism,

- 4) correct assembly of these elements,
- 5) proper service of braking systems during operation.

Diagnostic tests in Vehicle Inspection Stations allow to assess the technical condition of the braking system also in terms of assessing the quality of the construction elements. The purpose of diagnostic tests [1, 9, 19, 23, 24, 25] is to determine the state of the technical object or process at the moment considered as important. This is necessary to make a judgement about the service ability or inability of a technical object by comparing the actual state - instantaneous with the reference state, influencing the increase of safety and adequate efficiency of use [3]. The initial assessment of the braking system is aimed at determining the degree of wear of the system components and the causes of the identified symptoms of its disability (lack of technical suitability), which translates into the usable quality of the structure [6, 7, 8, 11, 12, 20]. The durability of devices in specific ambient conditions of the operating process depends mainly on the intensity of their use. Intensity of use is therefore the basic tool, with which it can influence on the damage process and the wear of the operational potential of technical facilities. Designing a braking system diagnosing unit for motor vehicles using the modelling techniques is therefore a matter of great importance. In particular when it concerns diagnostic problems used in automating exploitation processes [4].

## 2. Boolean model analysis of the leakage/hydraulic tightness testing process of the vehicle braking system $i \in 1..3$ , for and the HSUH target model in Boolean Algebra of Boolean functions. Technical interpretations

The research was undertaken in accordance with the recommendations of the morphological method. The phases and stages of the morphological method are included in Tab. 1. In contrast, Figure 1 presents a graphical model of "features - component objects" of the structure, describing the technical construction of HSUH in the next sorts of crosssections that are two, three, and five vectors, and ultimately in this paper - 14 components [4, 5, 9, 13, 15, 15, 16, 17, 18].

Phases	Stages
Problem determination	ldentifying the problem Defining the problem
Problem analysis	Identification of parameters Searching for possible parameter states
Synthesis	Construction of a morphological table Reduction of a morphological space

#### Table 1. Phases and stages of the morphological method



 $O_{HP}$  = hydraulic object of the front brakes,  $O_{HT}$  = hydraulic object of the rear brakes,  $P_{H}$  = breaks pump,  $P_{HP}$  = hydraulic brake subsystem of front brakes,  $P_{HT}$  = hydraulic brake subsystem of rear brakes,  $PP_{HP}$  = right hydraulic brake subsystem of front brakes,  $LP_{HP}$  = left hydraulic brake subsystem of front brakes,  $PP_{HT}$  = right hydraulic brake subsystem of rear brakes,  $LP_{HT}$  = left hydraulic brake subsystem of front brakes,  $PP_{HT}$  = right hydraulic brake subsystem of rear brakes,  $LP_{HT}$  = left hydraulic brake subsystem of rear brakes,  $UTT_{PH}$  = rear brake pump piston seals,  $K_{PH}$  = brake pump body,  $UTPP_{CH}$  = right piston seals of front brake cylinder,  $KPP_{CH}$  = right front brake cylinder body,  $PHPP_{CHP}$  = brake line connecting the right front brake cylinder with the master cylinder,  $UTPT_{CH}$  = right piston seals of the rear brake cylinder,  $KPT_{CH}$  = right rear cylinder body brake,  $PHPT_{CHP}$  = brake line connecting the right front brake cylinder with the master cylinder,  $UTPT_{CHP}$  = brake line connecting the right rear cylinder, with the master cylinder,  $UTPT_{CHP}$  = brake line connecting the right rear cylinder with the master cylinder,  $UTPT_{CHP}$  = brake line connecting the right rear cylinder with the master cylinder,  $PHPT_{CHP}$  = brake line connecting the right rear cylinder with the master cylinder,  $VTLT_{CH}$  = left piston seals of the rear brake cylinder,  $KLT_{CH}$  = left rear brake cylinder body,  $PHLT_{CHP}$  = brake line connecting the right rear cylinder with the master cylinder,  $UTLT_{CH}$  = left piston seals of the rear brake cylinder,  $KLT_{CH}$  = left rear brake cylinder body,  $PHLT_{CHP}$  = brake line connecting the right rear cylinder with the master cylinder body,  $PHLT_{CHP}$  = brake line connecting the left rear brake cylinder body,  $PHLT_{CHP}$  = brake line connecting the left rear brake cylinder body,  $PHLT_{CHP}$  = brake line connecting the left rear brake cylinder body,  $PHLT_{CHP}$  = brake line connecting

When testing based on the morphological analysis, one should proceed to the synthesis phase of parameters (from Boolean variables to Boolean functions), possible states (from Boolean algebra of function arguments to Boolean algebra of Boolean functions and synthesis of relations that this phase is described by actions on Boolean functions formulated in the following models  $HSUH_i$ ,  $i \in 1...3$  and HSUH.

Boolean algebra of Boolean functions k-variables for these models,

where:

$$(i,k) \in \{(1,2); (2,3); (3,5); (4,14)\}$$

have the form:

$$B_{(i,k)} = (FB_{(i,k)}, +, \cdot, \overline{(\cdot)}, 0, 1)$$

 $FB_{(i,k)}$  = set of Boolean functions k-variables for HSUH, models,

 $+,,,\overline{(\cdot)}-$  actions on Boolean functions from the sets  $FB_{(i,k)}$ 

 $f_{(i,k)} = 0$ ,  $f_{(i,k)} = 1 - \text{zero functions and one functions in the set } FB_{(i,k)}$ 

### 2.1. HSUH, model in Boolean algebra of Boolean functions

Let:

$$r = (r_1, r_2) \in B^2 \wedge f_1 : B^2 \rightarrow B$$

 $f_1(\mathbf{r}) = r_1 \cdot r_2$  – HSUH<sub>1</sub> leakage function and

 $f_{1OP}(\mathbf{r}) = r_1 - \text{Boolean leakage function } f_{1OP} : B^2 \to B \quad O_{HP}, \ \overline{f_{1OP}}(\mathbf{r}) = \overline{r_1} = f_{1SOP}(\mathbf{r}) - \text{Boolean tightness function}, O_{HP}$ 

 $\begin{array}{l} f_{1OT}(\mathbf{r}) = r_2 - \text{Boolean leakage function } f_{1OT} : B^2 \rightarrow B \quad O_{HT}, \\ \hline \hline \\ \hline \hline \\ f_{1OT}(\mathbf{r}) = r_2 = f_{1SOT}(\mathbf{r}) - \text{Boolean function, } O_{HT}. \end{array}$ 

 $f_1: B^2 \rightarrow B \ f_{1OP} \ f_{1OT}$  Boolean leakage function of  $\text{HSUH}_1$  model is the product of Boolean function and the form:

$$f_1(\mathbf{r}) = f_{1OP}(\mathbf{r}) \cdot f_{1OT}(\mathbf{r})$$
$$\overline{f_1(\mathbf{r})} = \overline{f_{1OP}(\mathbf{r})} + \overline{f_{1OT}(\mathbf{r})} = f_{1SOP}(\mathbf{r}) + f_{1SOT}(\mathbf{r}) = f_{1S}(\mathbf{r})$$

 $f_{1S}: B^2 \rightarrow B \quad f_{1SOP} \quad f_{1SOT}$  and Boolean tightness function of  $\text{HSUH}_1$  model is the sum of Boolean functions and the form:

$$\overline{f_1(\mathbf{r})} = \overline{f_{1OP}(\mathbf{r}) \cdot f_{1OT}(\mathbf{r})} = \overline{f_{1OP}(\mathbf{r})} + \overline{f_{1OT}(\mathbf{r})} = f_{1SOP}(\mathbf{r}) + f_{1SOT}(\mathbf{r}) = f_{1S}(\mathbf{r})$$

In the Boolean algebra language of Boolean functions, this model formulates the structural information of the global diagnosis of leakage/tightness  $O_{_{HP}}$  and  $O_{_{HT}}$ .

l.e.:

 $f_1(\mathbf{r}) = f_{1OP}(\mathbf{r}) \cdot f_{1OT}(\mathbf{r})$  – Leakage in HSUH<sub>1</sub> model is the product of Boolean leakage of hydraulic brake subsystems of front and rear brakes and

 $f_{1S}(\mathbf{r}) = f_{1SOP}(\mathbf{r}) + f_{1SOT}(\mathbf{r})$  – tightness in HSUH<sub>1</sub> model is the sum of Boolean leakage of hydraulic brake subsystems of front and rear brakes.

### 2.2 HSUH, model in Boolean algebra of Boolean functions

Let:

 $\mathbf{s} = (s_1, s_2, s_3) \in B^3$   $f_2 : B^3 \to B$  – and Boolean leakage function of  $\mathrm{HSUH}_2$  model. Because:

$$f_2(\mathbf{s}) = (s_1 + s_2)(s_1 + s_3)$$

this defining Boolean functions:

 $f_{_{2PP}}(\mathbf{s}) = s_{_{1}} + s_{_{2}}; f_{_{2PP}}: B^{^{2}} \rightarrow B$  – Boolean leakage function  $P_{_{H}}$  or  $P_{_{HP}}$ 

 $f_{2PT}(\mathbf{s}) = s_1 + s_3; f_{2PT} : B^2 \rightarrow B$  – Boolean leakage function  $P_H$  or  $P_{HT}$ 

it follows that Boolean function  $f_2$  is the product of Boolean functions of the form:

$$f_2(\mathbf{s}) = f_{2PP}(\mathbf{s}) \cdot f_{2PT}(\mathbf{s})$$

Accepting, in turn, in a natural way that further Boolean functions can be defined as follows:

 $f_{2PH}: B^2 \to B$  and  $f_{2PH}(\mathbf{s}) = s_1$  – Boolean leakage function *PH*,

 $f_{2OP}: B^2 \to B$  and  $f_{2OP}(\mathbf{s}) = s_2$  – Boolean leakage function  $P_{\mu\nu}$ 

 $f_{2OT}: B^2 \to B$  and  $f_{2OT}(\mathbf{s}) = s_3$  – Boolean leakage function  $P_{HT}$ 

and from the properties of Boolean algebra:

$$f_2(\mathbf{s}) = (s_1 + s_2)(s_1 + s_3) = s_1 + s_2 s_3$$
 (MAPN of the functions  $f_2$ )

the following is obtained in Boolean algebra of Boolean functions:

$$f_2(\mathbf{s}) = f_{2PH} + f_{2OP} \cdot f_{2OT}$$

In technical interpretation, it follows that the failure of  $\mathrm{HSUH}_2$  occurs if and only if the master cylinder is leaky *PH* or at the same time both hydraulic brake subsystems  $P_{_{HP}}$  and  $P_{_{HT}}$  front and rear brakes.

PH Failure of  $\mathrm{HSUH}_2$  is Boolean sum of the failure of the master cylinderand Boolean product of leakage of hydraulic brake subsystems of front and rear brakes.

Thus, on the basis of de Morgan's laws, the tightness function  $f_{2S}: B^3 \rightarrow B$  takes the form of:

$$\overline{f_{2}(\mathbf{s})} = f_{2S}(\mathbf{s}) = \overline{f_{2PH} + f_{2OP} \cdot f_{2OT}} = \begin{bmatrix} \overline{a+b} = \overline{a} \cdot \overline{b} \\ \overline{ab} = \overline{a} + \overline{b} \end{bmatrix} = \overline{f_{2PH}(\mathbf{s})} \cdot (\overline{f_{2OP}(\mathbf{s})} + \overline{f_{2OT}(\mathbf{s})}) =$$

$$= \overline{f_{2PH}(\mathbf{s})} \cdot \overline{f_{2OP}(\mathbf{s})} + \overline{f_{2PH}(\mathbf{s})} \cdot \overline{f_{2OT}(\mathbf{s})} = \begin{bmatrix} \overline{f_{2PH}} = f_{2SPH} - \text{tightness function } PH(f_{2SPH}(\mathbf{s}) = \overline{s_{1}}) \\ \overline{f_{2OP}} = f_{2SOP} - \text{tightness function } P_{HP}(f_{2SOP}(\mathbf{s}) = \overline{s_{2}}) \\ \overline{f_{2OT}} = f_{2SOT} - \text{fightness function } P_{HT}(f_{2SOT}(\mathbf{s}) = \overline{s_{3}}) \end{bmatrix} =$$

$$= f_{2SPH}(\mathbf{s}) \cdot f_{2SOP}(\mathbf{s}) + f_{2SPH}(\mathbf{s}) \cdot f_{2SOT}(\mathbf{s})$$

In the technical interpretation of the function  $f_{\rm 2S}$ , the efficiency of  ${\rm HSUH_2}$  is determined by the efficiency of the master cylinder PH and the hydraulic brake subsystem of the front brakes  $P_{\rm HP}$  or at the same time the efficiency of the master cylinder PH and the hydraulic brake sub-system of rear brakes  $P_{\rm HP}$ .

The efficiency of  $\mathrm{HSUH}_2$  is Boolean sum of Boolean product of the efficiency of the master cylinder by the efficiency of the hydraulic brake subsystem of front brakes and by the efficiency of the hydraulic brake subsystem of rear brakes, respectively.

#### 2.3. HSUH, model in Boolean algebra of Boolean functions

Let:

$$\begin{split} \mathbf{u} &= (u_1, u_2, u_3, u_4, u_5) \in B^5 \quad \text{and} \quad f_3 : B^5 \to B \quad \text{- Boolean leakage function of the model,} \\ \text{HSUH}_3 &= (PH, (PP_{\text{\tiny HP}}, LP_{\text{\tiny HP}}), (PP_{\text{\tiny HT}}, LP_{\text{\tiny HT}})) \end{split}$$

Because:

$$f_3(\mathbf{u}) = (u_1 + u_2 + u_3)(u_1 + u_4 + u_5)$$

this defining Boolean functions:

 $f_{_{3PP}}(\mathbf{u}) = u_1 + u_2 + u_3; f_{_{3PP}} : B^5 \rightarrow B$  – Boolean leakage function  $P_H$  or  $PP_{_{HP}}$  or  $LP_{_{HP}}$ 

 $f_{_{3PT}}(\mathbf{u}) = u_1 + u_4 + u_5; f_{_{3PT}} : B^5 \rightarrow B$  – Boolean leakage function  $P_{_{HT}}$  or  $PP_{_{HT}}$  or  $LP_{_{HT}}$ 

it follows that Boolean function  $f_3$  is the product of Boolean functions of the form:

$$f_3(\mathbf{u}) = f_{_{3PP}}(\mathbf{u}) \cdot f_{_{3PT}}(\mathbf{u})$$

Accepting, in turn, in a natural way that further Boolean functions can be defined as follows:

 $f_{_{3PH}}: B^{_{5}} \rightarrow B$  and  $f_{_{3PH}}(\mathbf{u}) = u_1$  – Boolean leakage function PH,

 $f_{_{3OP}}: B^s \to B$  and  $f_{_{3OP}}(\mathbf{u}) = u_2 + u_3$  – Boolean leakage function in the hydraulic brake system of front axis, i.e.  $PP_{_{HP}}$  or  $LP_{_{HP'}}$ 

 $f_{_{3OPP}}, f_{_{3OPL}}: B^{_{5}} \rightarrow B$  and  $f_{_{3OPP}}(\mathbf{u}) = u_2, f_{_{3OPL}}(\mathbf{u}) = u_3$  – Boolean leakage functions, respectively,  $PP_{_{HP}}$  and  $LP_{_{HP}}$  and  $f_{_{3OP}}(\mathbf{u}) = f_{_{3OPP}}(\mathbf{u}) + f_{_{3OPL}}(\mathbf{u})$ 

 $f_{3or}: B^s \to B$  and  $f_{3or}(\mathbf{u}) = u_4 + u_5$  – Boolean leakage functions in the hydraulic brake system of rear axis, i.e  $PP_{\mu r}$  and  $LP_{\mu r}$ 

 $f_{3OTP}, f_{3OTL}: B^5 \rightarrow B$  and  $f_{3OTP}(\mathbf{u}) = u_4, f_{3OTL}(\mathbf{u}) = u_5$  – Boolean leakage function respectively  $PP_{HT}$  and  $LP_{HT}$ .

From the properties of Boolean algebra:

$$f_3(\mathbf{u}) = (u_1 + u_2 + u_3)(u_1 + u_4 + u_5) = u_1 + (u_2 + u_3)(u_4 + u_5)$$
 (MAPN of the functions  $f_3$ )

5 variables are obtained in Boolean algebra of Boolean functions:

$$f_3(\mathbf{u}) = f_{3PH} + f_{3OP} \cdot f_{3OT}$$

 $\mathrm{HSUH}_3$  leakage is Boolean sum of the brake cylinder leak function and the product of Boolean hydraulic leakage function of the brake system of front axis by the leak functions of the hydraulic braking system of rear axis.

The technical interpretation of the function  $f_3: B^s \to B$  results from the leakage of HSUH<sub>3</sub> if and only if the master cylinder is leaking or *PH* at the same time the left or right hydraulic brake subsystem of front brakes is leaking  $LP_{HP}$ ,  $PP_{HP}$  and the hydraulic brake subsystem of rear brakes left or right  $LP_{HT}$ ,  $PP_{HT}$ .

Taking into account the more detailed morphological space described in the sequence,  $HSUH_3 = (PH, PP_{HP}, LP_{HP}, LP_{HT}, LP_{HT})$  a more accurate description of leakage is obtained in Boolean algebra, the Boolean functions are written on all sides of the front and rear axes.

$$f_{3}(\mathbf{u}) = f_{3PH}(\mathbf{u}) + f_{3OP}(\mathbf{u}) \cdot f_{3OT}(\mathbf{u}) = \begin{bmatrix} f_{3OP}(\mathbf{u}) = f_{3OPP}(\mathbf{u}) + f_{3OPL}(\mathbf{u}) \\ f_{3OT}(\mathbf{u}) = f_{3OTP}(\mathbf{u}) + f_{3OTL}(\mathbf{u}) \end{bmatrix} = \\ = f_{3PH}(\mathbf{u}) + (f_{3OPP}(\mathbf{u}) + f_{3OPL}(\mathbf{u})) \cdot (f_{3OTP}(\mathbf{u}) + f_{3OTL}(\mathbf{u})) = [a(b+c) = ab + ac] = \\ = f_{3PH}(\mathbf{u}) + f_{3OPP}(\mathbf{u}) \cdot f_{3OTP}(\mathbf{u}) + f_{3OPP}(\mathbf{u}) \cdot f_{3OTP}(\mathbf{u}) + f_{3OPL}(\mathbf{u}) \cdot f_{3OTL}(\mathbf{u})$$

In the technical interpretation of the leakage function in the form  $f_3: B^s \to B$  of five components, it results that the leakage of  $\mathrm{HSUH}_3$  is observed if and only if the master cylinder is leaking and at least the two components are leaking from the following two-components collections:  $\{PP_{\mu\nu}, PP_{\mu\tau}\}, \{PP_{\mu\nu}, LP_{\mu\tau}\}, \{LP_{\mu\nu}, PP_{\mu\tau}\}, \{LP_{\mu\nu}, LP_{\mu\tau}\}$ . Continuing the function transformation  $f_3$  (on the basis of de Morgan's laws), the tightness function for  $\mathrm{HSUH}_3$  model is obtained:

$$\overline{f_{3}(\mathbf{u})} = f_{3S}(\mathbf{u}) = \overline{f_{3PH}(\mathbf{u}) + f_{3OP}(\mathbf{u}) \cdot f_{3OT}(\mathbf{u})} = \begin{bmatrix} \overline{a+b} = \overline{a} \cdot \overline{b} \\ \overline{ab} = \overline{a} + \overline{b} \end{bmatrix} = \overline{f_{3PH}(\mathbf{u})} \cdot (\overline{f_{3OP}(\mathbf{u})} + \overline{f_{3OT}(\mathbf{u})}) = \\ = \begin{bmatrix} a(b+c) = ab + ac \end{bmatrix} = \overline{f_{3PH}(\mathbf{u})} \cdot \overline{f_{3OP}(\mathbf{u})} + \overline{f_{3PH}(\mathbf{u})} \cdot \overline{f_{3OT}(\mathbf{u})} = \\ \begin{bmatrix} \overline{f_{3PH}} = f_{3SPH} - \text{tightness function } PH(f_{3SPH}(\mathbf{u}) = \overline{u_{1}}) \\ \hline \overline{f_{3OP}} = f_{3SOP} - \text{tightness function } PP_{HP} \text{ and } LP_{HP}(f_{3SOP}(\mathbf{u}) = \overline{u_{2}u_{3}}) \\ \hline \overline{f_{3OT}} = f_{3SOT} - \text{tightness function } PP_{HT} \text{ and } LP_{HT}(f_{3SOT}(\mathbf{u}) = \overline{u_{4}u_{5}}) \end{bmatrix} = \\ = f_{3SPH}(\mathbf{u}) \cdot f_{3SOP}(\mathbf{u}) + f_{3SPH}(\mathbf{u}) \cdot f_{3SOT}(\mathbf{u})$$

In the technical interpretation of Boolean function  $f_{3s}: B^s \to B$ , it can be said that the hydraulic brake system in HSUH<sub>3</sub> is tight if and only if the brake cylinder is tight and the hydraulic brake subsystem of front axis is leak at the same time or there is the master cylinder is tight and, at the same time, the hydraulic brake subsystem of rear axis.

Efficiency in  $\mathrm{HSUH}_3$  model is Boolean sum of Boolean products of the brake pump tightness function by the tightness function of the hydraulic beake subsystem of front and rear axes.

Going to the next version of the detail of the morphological space describing  $HSUH_{3}$ , the following form of the leakage function for this model is obtained:

$$\overline{f_{3S}(\mathbf{u})} = \overline{f_{3PH}(\mathbf{u}) + (f_{3OPP}(\mathbf{u}) + f_{3OPL}(\mathbf{u})) \cdot (f_{3OTP}(\mathbf{u}) + f_{3OTL}(\mathbf{u}))} = \begin{bmatrix} \overline{a + b} = \overline{a} \cdot \overline{b} \\ \overline{ab} = \overline{a} + \overline{b} \end{bmatrix} =$$

$$= \overline{f_{3PH}(\mathbf{u})} \cdot (\overline{(f_{3OPP}(\mathbf{u}) + f_{3OPL}(\mathbf{u}))} + \overline{(f_{3OTP}(\mathbf{u}) + f_{3OTL}(\mathbf{u}))}) =$$

$$= \overline{f_{3PH}(\mathbf{u})} \cdot (f_{3OPP}(\mathbf{u}) \cdot \overline{f_{3OPL}(\mathbf{u})} + f_{3OTP}(\mathbf{u}) \cdot \overline{f_{3OTL}(\mathbf{u})}) =$$

$$= \overline{f_{3PH}(\mathbf{u})} \cdot \overline{f_{3OPP}(\mathbf{u})} \cdot \overline{f_{3OPL}(\mathbf{u})} + \overline{f_{3PH}(\mathbf{u})} \cdot \overline{f_{3OTL}(\mathbf{u})} =$$

$$= \begin{bmatrix} \overline{f_{3OPP}} = f_{3SOPP} - \text{tightness function } PP_{HP}, \overline{f_{3OTL}} = f_{3SOTP} - \text{tightness function } PP_{HT} \\ \overline{f_{3OPL}} = f_{3SOPL} - \text{tightness function } LP_{HP}, \overline{f_{3OTL}} = f_{3SOTL} - \text{tightness function } LP_{HT} \end{bmatrix} =$$

$$= f_{SPH}(\mathbf{u}) \cdot f_{3OPP}(\mathbf{u}) \cdot f_{3OPL}(\mathbf{u}) + f_{SPH}(\mathbf{u}) \cdot f_{3SOTP}(\mathbf{u}) \cdot f_{3SOTL}(\mathbf{u})$$

In technical interpretation from the form:

$$f_{3S}(\mathbf{u}) = f_{3SPH}(\mathbf{u}) \cdot f_{3SOPP}(\mathbf{u}) \cdot f_{3SOPL}(\mathbf{u}) + f_{3SPH}(\mathbf{u}) \cdot f_{3SOTP}(\mathbf{u}) \cdot f_{3SOTL}(\mathbf{u})$$

Boolean functions of 5 variables in Boolean algebra show that the tightness of  $\text{HSUH}_3$  guarantees the tightness of all objects from the sequence (*PH*, *PP*<sub>*HP*</sub>, *LP*<sub>*HP*</sub>) or from the sequence (*PH*, *PP*<sub>*HP*</sub>, *LP*<sub>*HP*</sub>).

#### 2.4. HSUH model in Boolean algebra of Boolean functions

Let:

 $\mathbf{x} = (x_i)_{i=1}^{14} \in B^{14}$  and  $f: B^{14} \to B$  – Boolean leakage function of HSUH Boolean model.

Because:

$$f(\mathbf{x}) = \sum_{i=1}^{8} x_i \cdot (x_1 + x_2 + \sum_{j=9}^{14} x_j)$$

this defining Boolean functions:

 $f_{_{PP}}(\mathbf{x}) = \sum_{i=1}^{8} x_i$ ;  $f_{_{PP}}: B^{_{14}} \rightarrow B$  – Boolean function of leakage of components  $UTT_{_{PH}}$  and  $K_{_{PH}}$  the master cylinder and the components of the hydraulic brake system of front axis  $UTPP_{_{CHP}}$ ;  $KPP_{_{CHP}}$   $PHPP_{_{CHP}}$   $UTLP_{_{CHP}}$   $KLP_{_{CHP}}$   $PHLP_{_{CHP}}$ 

 $f_{PT}(\mathbf{x}) = x_1 + x_2 + \sum_{j=9}^{14} x_j$ ;  $f_{PT}: B^{14} \rightarrow B$  – Boolean function of leakage of the components  $UTT_{PH}$  and  $K_{PH}$  the master cylinder as well as the components of the hydraulic brake system of rear axis  $UTPT_{CH'} KPT_{CH'} PHPT_{CHP} UTLT_{CH'} KLT_{CH'} PHLT_{CHP}$  it follows that Boolean function f is the product of Boolean functions  $f_{PP}$  and  $f_{PT}$  in their Boolean algebra, Boolean functions of 14 variables

$$f(\mathbf{x}) = f_{PP}(\mathbf{x}) \cdot f_{PT}(\mathbf{x})$$

Continuing Boolean transformations of the form defining Boolean function  $f: B^{14} \rightarrow B$ 

$$f(\mathbf{x}) = (x_1 + x_2 + \sum_{i=3}^{8} x_i)(x_1 + x_2 + \sum_{j=9}^{14} x_j) = \begin{bmatrix} a(b+c) = ab + ac \\ aa = a \\ a + ab = a \end{bmatrix} = x_1 + x_2 + \sum_{i=3}^{8} x_i \cdot \sum_{j=9}^{14} x_j$$

and formulating, in a natural way, the resulting forms, further Boolean functions:

 $f_{PH}: B^{14} \to B$  and  $f_{PH}(\mathbf{x}) = x_1 + x_2$  – Boolean function of leakage of the master cylinder  $P_H$  in HSUH model,

 $f_{OP}: B^{14} \to B$  and  $f_{OP}(\mathbf{x}) = \sum_{i=3}^{8} x_i$  – Boolean function of leakage of all components of the hydraulic brake subsystem in the brake system of front axis

 $f_{OPP}, f_{OPL}: B^{14} \to B, f_{OPP}(\mathbf{x}) = \sum_{i=3}^{5} x_i, f_{OPL}(\mathbf{x}) = \sum_{j=6}^{8} x_j$  – Boolean leakage functions respectively in the front axis subsystem of the hydraulic brake system:

$$(UTPP_{CH}, KPP_{CH}, PHPP_{CHP})$$
 and  $(UTLP_{CH}, KLP_{CH}, PHLP_{CHP})$ 

 $f_{OTP}, f_{OTL}: B^{14} \to B, f_{OTP}(\mathbf{x}) = \sum_{i=9}^{11} x_i, f_{OTL}(\mathbf{x}) = \sum_{j=12}^{14} x_j$  – Boolean leakage functions respectively in the rear axis subsystem of the hydraulic brake system:

$$(UTPT_{CH}, KPT_{CH}, PHPT_{CHP})$$
 and  $(UTLT_{CH}, KLT_{CH}, PHLT_{CH})$ ,

 $f_{OT}: B^{14} \to B$  and  $f_{OT}(\mathbf{x}) = \sum_{j=9}^{14} x_j$  – Boolean leakage function of all components of the hydraulic subsystem in the brake system of rear axis is obtained in Boolean algebra of Boolean functions:

$$f(\mathbf{x}) = f_{PH}(\mathbf{x}) + f_{OP}(\mathbf{x}) \cdot f_{OT}(\mathbf{x})$$

So Boolean leakage function in Boolean HSUH model is Boolean sum of the leakage function of the  $f_{PH}$  master cylinder and Boolean product of two Boolean leakage functions of the hydraulic brake system, respectively front axis and rear axis.

Taking into account the more detailed morphological space described by HSUH sequence, a more accurate description of leakage in Boolean algebra of Boolean functions is obtained:

$$f(\mathbf{x}) = f_{3PH}(\mathbf{x}) + f_{OP}(\mathbf{x}) \cdot f_{OT}(\mathbf{x}) = \begin{bmatrix} f_{OP}(\mathbf{x}) = f_{OPP}(\mathbf{x}) + f_{OPL}(\mathbf{x}) \\ f_{OT}(\mathbf{x}) = f_{OTP}(\mathbf{x}) + f_{OPL}(\mathbf{x}) \end{bmatrix} =$$
$$= f_{PH}(\mathbf{x}) + (f_{OPP}(\mathbf{x}) + f_{OPL}(\mathbf{x})) \cdot (f_{OTP}(\mathbf{x}) + f_{OTL}(\mathbf{x})) = [a(b+c) = ab + ac] =$$
$$= f_{PH}(\mathbf{x}) + f_{OPP}(\mathbf{x}) \cdot f_{OTP}(\mathbf{x}) + f_{OPP}(\mathbf{x}) \cdot f_{OTP}(\mathbf{x}) + f_{OPL}(\mathbf{x}) \cdot f_{OTL}(\mathbf{x})$$

The technical interpretation of this form in Boolean algebra of Boolean functions takes into account both leakages of the master cylinder as well as leakage of the relevant components regarding the proper axes and their sides.

The forms in Boolean algebra of Boolean functions of 14 Boolean variables define the more global structure of leakage detection. Continuing the transformation in Boolean algebra of 14 variables Boolean functions, the following is obtained:

$$\overline{f(\mathbf{x})} = f_{\mathcal{S}}(\mathbf{x}) = \overline{f_{3PH}(\mathbf{x}) + f_{OP}(\mathbf{x}) \cdot f_{OT}(\mathbf{x})} = \begin{bmatrix} \overline{f_1 + f_2} = \overline{f_1} \cdot \overline{f_2} \\ \overline{f_1 f_2} = \overline{f_1} + \overline{f_2} \end{bmatrix} = \overline{f_{PH}(\mathbf{x})} \cdot (\overline{f_{OP}(\mathbf{x})} + \overline{f_{OT}(\mathbf{x})}) = \\ = \begin{bmatrix} f_1(f_2 + f_3) = f_1 f_2 + f_1 f_3 \end{bmatrix} = \overline{f_{PH}(\mathbf{x})} \cdot \overline{f_{OP}(\mathbf{x})} + \overline{f_{PH}(\mathbf{x})} \cdot \overline{f_{OT}(\mathbf{x})} = \\ \\ \begin{bmatrix} \overline{f_{PH}(\mathbf{x})} = \overline{\mathbf{x}_1 + \mathbf{x}_2} = \overline{\mathbf{x}_1} \cdot \overline{\mathbf{x}_2} = f_{SPH}(\mathbf{x}) - & \text{Boolean function of tightness and components} \\ \text{of the master cylinder construction} \\ \\ \hline \overline{f_{OP}(\mathbf{x})} = \overline{\sum_{i=3}^8 \overline{x_i}} = \sum_{i=3}^8 \overline{x_i} = f_{SOP}(\mathbf{x}) - & \text{Boolean function of tightness and components} \\ \text{of the front axis of the brake system} \\ \\ \hline \overline{f_{OT}(\mathbf{x})} = \overline{\sum_{i=3}^1 \mathbf{x}_i} = \sum_{j=9}^1 \overline{\mathbf{x}_j} = f_{SOT}(\mathbf{x}) - & \text{Boolean function of tightness and components} \\ \text{of the rear axis of the brake system} \\ \\ = \begin{bmatrix} f_{SPH}(\mathbf{x}) \cdot f_{SOP}(\mathbf{x}) + f_{SPH}(\mathbf{x}) \cdot f_{SOT}(\mathbf{x}) \\ \end{bmatrix}$$

So for:

$$\mathbf{x} = (x_i)_{i=1}^{14} \land x_i \in B \land f_S : B^{14} \to B$$

it is the tightness function in HSUH Boolean model and:

$$f_{S}(\mathbf{x}) = f_{SPH}(\mathbf{x}) \cdot f_{SOP}(\mathbf{x}) + f_{SPH}(\mathbf{x}) \cdot f_{SOT}(\mathbf{x})$$

The tightness function in Boolean HSUH model is Boolean sum of Boolean products of Boolean function of the master cylinder tightness by the tightness function of front axis and by the tightness function of the hydraulic brake system of rear axis.

Going to the next version of the detail of the morphological space for HSUH, the following form of the leakage function is obtained:

$$\overline{f(\mathbf{x})} = \overline{f_{s}(\mathbf{u})} = \overline{f_{PH}(\mathbf{u}) + (f_{OPP}(\mathbf{u}) + f_{OPL}(\mathbf{u})) \cdot (f_{OTP}(\mathbf{u}) + f_{OTL}(\mathbf{u}))} = \begin{bmatrix} \overline{a + b} = \overline{a \cdot b} \\ \overline{ab} = \overline{a} + \overline{b} \end{bmatrix} =$$

$$= \overline{f_{PH}(\mathbf{x})} \cdot (\overline{(f_{OPP}(\mathbf{x}) + f_{OPL}(\mathbf{x}))} + \overline{(f_{OTP}(\mathbf{x}) + f_{OTL}(\mathbf{x}))}) = \overline{f_{PH}(\mathbf{x})} \cdot (\overline{f_{OPP}(\mathbf{x})} \cdot \overline{f_{OPL}(\mathbf{x})} +$$

$$+ \overline{f_{OTP}(\mathbf{x})} \cdot \overline{f_{OTL}(\mathbf{x})}) = \begin{bmatrix} \overline{f_{PH}(\mathbf{x})} = f_{SPH}(\mathbf{x}) - \text{ tightness function } - PH \\ \overline{f_{OPP}(\mathbf{x})} = f_{SOPP}(\mathbf{x}) - \text{ tightness function } - PP_{HP} \\ \overline{f_{OPL}(\mathbf{x})} = f_{SOPL}(\mathbf{x}) - \text{ tightness function } - LP_{HP} \\ \hline \overline{f_{OTP}(\mathbf{x})} = f_{SOTP}(\mathbf{x}) - \text{ tightness function } - PP_{HT} \\ \overline{f_{OTL}(\mathbf{x})} = f_{SOTL}(\mathbf{x}) - \text{ tightness function } - LP_{HT} \\ \end{bmatrix} =$$

$$= f_{SPH}(\mathbf{x}) \cdot f_{SOPP}(\mathbf{x}) \cdot f_{OPL}(\mathbf{x}) + f_{SPH}(\mathbf{x}) \cdot f_{SOTP}(\mathbf{x}) \cdot f_{SOTL}(\mathbf{x})$$

l.e.:

$$f_{S}(\mathbf{x}) = f_{SPH}(\mathbf{x}) \cdot f_{SOPP}(\mathbf{x}) \cdot f_{OPL}(\mathbf{x}) + f_{SPH}(\mathbf{x}) \cdot f_{SOTP}(\mathbf{x}) \cdot f_{SOTL}(\mathbf{x})$$

In the technical interpretation of this equation in Boolean algebra of Boolean functions, it can be concluded that the tightness of the hydraulic brake system in HSUH model is determined by the simultaneous tightness or all components of the master cylinder and the hydraulic brake subsystems on both sides of front axis or tightness of all components of the master cylinder and the hydraulic brake subsystems on both sides on both sides of rear axis.

Leakage of the hydraulic circuit of the service brake in motor vehicles causes the object to change from the technical condition to its unserviceability, which does not classify it for further use. As a result of this phenomenon, the effectiveness of braking forces is lost. A visual check of the brake fluid level and searching for leakage traces does not guarantee an effective control of the braking system condition, because in this way it is not possible to detect small leaks, especially those located in invisible places [24]. Therefore, an accurate assessment of its tightness is possible only with the help of special instruments in vehicles workshops or at the Vehicle Inspection Stations

The purpose of the conducted research was to develop a method for the process of diagnosing hydraulic circuit leaks for the needs of designing an onboard diagnosing unit of selected damages of the braking mechanisms. The mathematical model of the diagnostic process based on Boolean functions presented in the article is a part of the mathematical analysis and concerns the leakage testing process in the hydraulically controlled braking systems.

Based on the structure analysis, the basic components of the morphological space of three intermediate mathematical models of the structure of the diagnosis process and the target model of 14 components were developed. Boolean functions  $f, f_s: B^{14} \rightarrow B$  introduce a structure describing the natural initial diagnosis of HSUH from the point of view of leak-age/tightness of the master cylinder, outboard hydraulic gears of the front and rear axle and their both sides - left and right. Thus, they allow to analyse leakage/tightness problems in different cross-sections of detail selected during the preliminary tests of HSUH<sub>i</sub>  $i \in 1..3$ , models by decomposition of HSUH with the morphological method.

As a part of the design of the diagnostic system, sensors will be designed to detect, e.g. brake fluid pressure drop, signalling damage to the system, or the transition from the state of technical usefullness to non-usefullness. The system will also have a built-in diagnostic map designed for a selected group of vehicles based on the design and engineering scheme of the braking system and conducted mathematical analyses, informing the vehicle user about damage to the system and the place where the damage occurred.

The presented research results confirm the implementation of the purpose, as well as enable the formulation of guidelines for the further stage of analysis of the diagnosis structure and its minimization. The obtained structuring results may be used in the automation and robotics of the leakage/tightness of the vehicle hydraulic braking system.

The full text of the article is available in Polish online on the website http://archiwummotoryzacji.pl.

Tekst artykułu w polskiej wersji językowej dostępny jest na stronie http://archiwummotoryzacji.pl.

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